# Data Structures and Algorithms

## Lecture 03 – Sum of two numbers
## (Hashing, Algo complexity analysis, Sorting)



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# Problem Solving




**Which two suspects had a total of 12,000 in stolen money?**

# Solution 1

- Brute_force

```
def two_sum_brute_force(nums, target):
    n = len(nums)
    for i in range(n):
        for j in range(i + 1, n):
            if nums[i] + nums[j] == target:
                return [i, j]
    return []  # 无解
```



**Time Complexity**：？，**Space Complexity**：？

# Solution 2

- Hash_table

```
def two_sum_hash_table(nums, target):
    num_to_index = {}  # 哈希表：数值 -> 下标
    for i, num in enumerate(nums):
        complement = target - num
        if complement in num_to_index:
            return [num_to_index[complement], i]
        num_to_index[num] = i //哈希表建立的过程，判断补数complement是否存在于之前的Hashing中;
    return []  # 无解
```



Time Complexity：？，Space Complexity：？

# Solution 3

- Sorting + Dual pointers

```
def two_sum_two_pointers(nums, target):
    # 存储原始下标
    indexed_nums = [(num, i) for i, num in enumerate(nums)]
    indexed_nums.sort()  # 按数值排序
    left, right = 0, len(indexed_nums) - 1
    while left < right:
        num_left, idx_left = indexed_nums[left]
        num_right, idx_right = indexed_nums[right]
        current_sum = num_left + num_right
        if current_sum == target:
            return [idx_left, idx_right]
        elif current_sum < target:
            left += 1
        else:
            right -= 1
    return []  # 无解
```



**Time Complexity**：？，**Space Complexity**：？

# The Principal of Hash and its application

# Python Dictionaries

**A dictionary is a collection which is unordered, changeable and indexed**

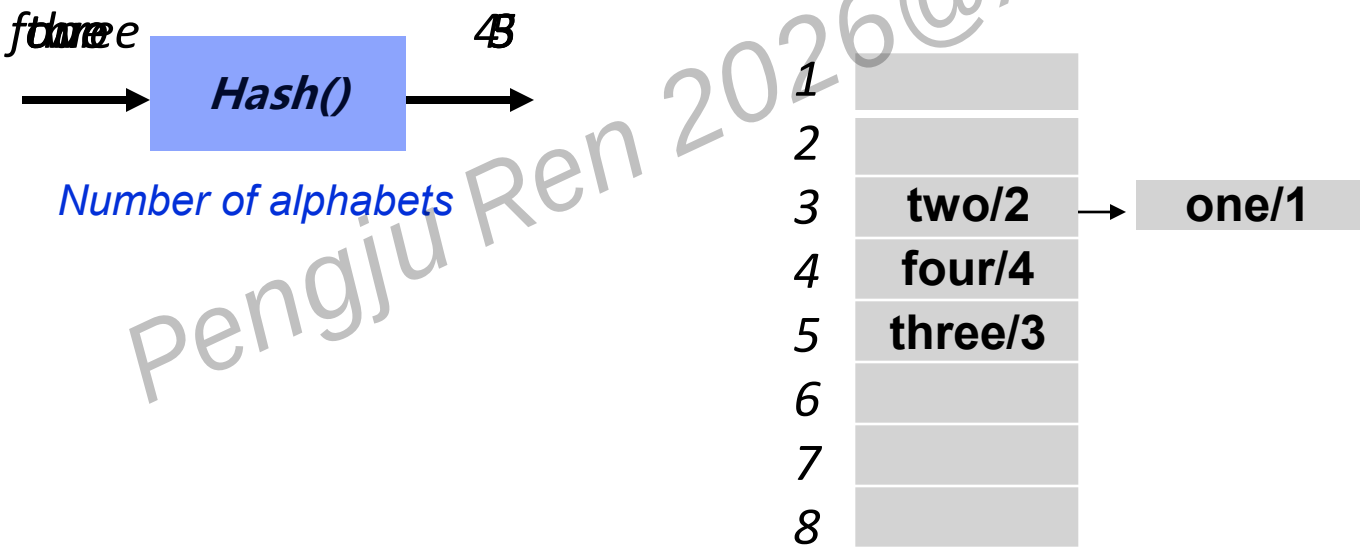

```
myDict = {“James”: ”GOAT of NBA”,
          “Curry”  : ”Best 3-point Shooter”}
myArray = [“James”, “Curry”]
```

```
myArray[1]          Curry
myDict[‘Curry’]     Best 3-point Shooter
```

# Hash table（Dictionary in Memory）

**A *hash table* is a way of doing *Key-value lookups,* you can store the *values* in an array, and then use a *hash function* to find the index of the array cell that corresponds to the *key-value pair*.**

Eng2Digit = [‘four’:4, ‘three’:3, ‘two’:2, ‘one’:1]






***Bad hash() function, too many conflicts !!!***

# What is Hashing

**Hashing is a method of allocating and indexing data. The idea behind hashing is to allow *large amounts of data* to be *indexed using keys* commonly created by formulas（hashing function）**



# Practical Use of Hashing

- **Password verification**

When you log into a website, how does the system confirm that your password is correct without storing your plaintext password?

- **File system: File path is mapped to physical location on disk**

When the number of files grows from dozens to billions, how do you quickly locate the corresponding file?

# Hashing Terminology

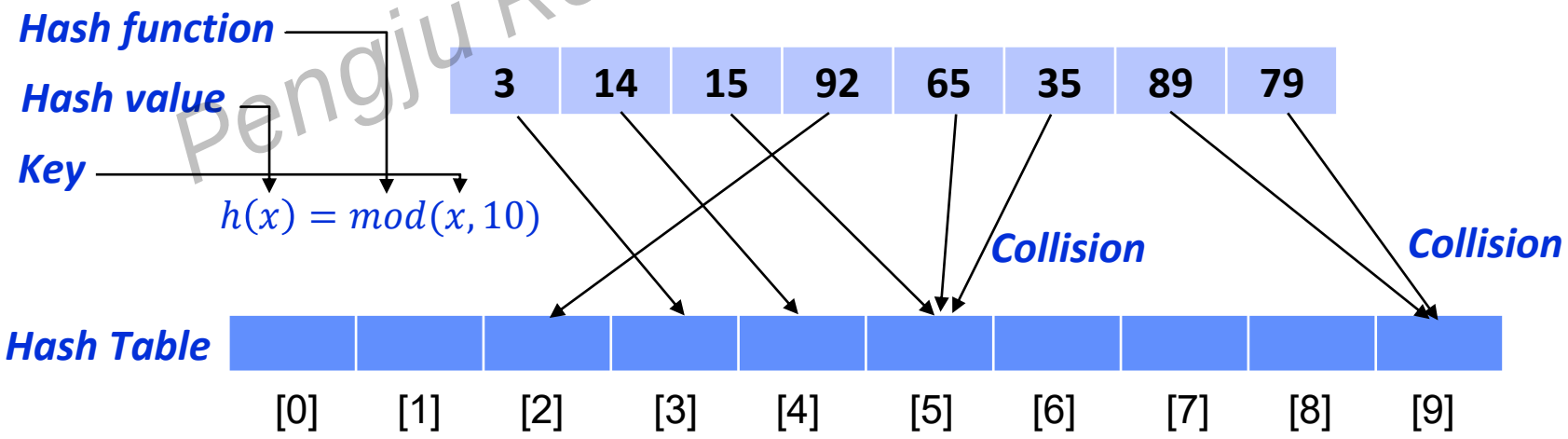
**Hash function:** It is a function that can be used to map of arbitrary size (n) to data of fixed size (m)

**Key:** Input data by user

**Hash value:** A value that is returned by Hash Function

**Hash Table:** It is a data structure which implements an associative array abstract data type, a structure that can map keys to value

**Collision:** A collision occurs when two different keys to a hash function produce the same output(hash value)



# Typical Hash Functions

**1. Direct Addressing Method**

Hash(key) = a * key + b (a, b are constants, a ≠ 0)

**2. Modulo Method**

Hash(key) = key mod m,  m is often chosen as a **prime number**

**3. Universal hash function**

$$\text{Hash}_{a,b}(\text{key}) = ((a*\text{key} + b) \bmod p) \bmod m$$

$$\Pr\{h(k_i) = h(k_j)\} = 1/m, \forall\, k_i \neq k_j \in \{0,1, \ldots n\}$$



**Good Hash function : computes quickly, distributes keys uniformly, and minimizes collisions**

# Example：Hash for Password verification

**Core Mechanism: One-Way Property and Avalanche Effect**

Registration Phase： User enters PW -> *fingerprint*= Hash(PW) -> send *fingerprint* to the website's database

Login: User enters PW -> hash value = Hash(PW) -> Compare with stored *fingerprint* at the website side



**One-way (irreversible):** You cannot reconstruct the original meat(*password*) from the mince(*hash value*).

**Avalanche Effect:** If you change just one letter in the original password, the resulting hash value will change drastically.

# Example：Hash for File Systems location

## Core Mechanism: Uniform Distribution & Accelerated Access

Taking a file's unique identifier (like its filename), compute its hash value (e.g., *77e1ba46ee3a2b2d1558d7c5d07c4c0caa46c7bf*), and then use the first few characters of this hash as names for multi-level subdirectories

For example, the path /77/e1/77e1ba46... works as follows:

1.Take *77* of the hash as the name of the first-level subdirectory.

2.Take *e1* as the name of the second-level subdirectory.

3.Use the complete hash value as the filename, stored under the */77/e1/* directory

# Collision Resolution Techniques

If deletion operations are frequent, we prefer this

## Direct Chaining

- **Hash table never gets full**
- **Huge linked list might causes performance leaks (Time complexity for search operation becomes O(n).)**

If the input size is known, we prefer this

## Open addressing

- **Easy implementation**
- **Linear probing:** place new key into closest following empty cell
- **Double hashing:** Interval between probes is computed by another hash function
- **When Hash table is full,** Create ***2X*** size of current Hash Table and recall hashing for current keys, creation of new Hash table affects performance (Time complexity for search operation becomes O(n).)

# Algorithm complexity analysis

# What makes a good Algorithm?

**Correctness & Efficiency (both in time and space)**

**$O(\cdot)$、$\Omega(\cdot)$、$\Theta(\cdot)$ are used to describe the efficiency of algorithms**

**Time complexity: A way of showing how the runtime of a function increases as the size of the input increases.**



**The function f(n)=O(g(n)), iff $\exists\ c\ and\ n_0$, such that f(n)≤cg(n) , $\forall n \geq n_0$**

**The function f(n)=Ω(g(n)), iff $\exists\ c\ and\ n_0$, such that f(n)≥cg(n) , $\forall n \geq n_0$**

**The function f(n)=Θ(g(n)), iff $\exists\ c_1, c_2\ and\ n_0$, such that $c_1$g(n)≥f(n)≥ $c_2$g(n) , $\forall n \geq n_0$**

# What makes a good Algorithm?

**Correctness & Efficiency (both in time and space)**

**$O(\cdot)$、$\Omega(\cdot)$、$\Theta(\cdot)$ are used to describe the efficiency of algorithms**

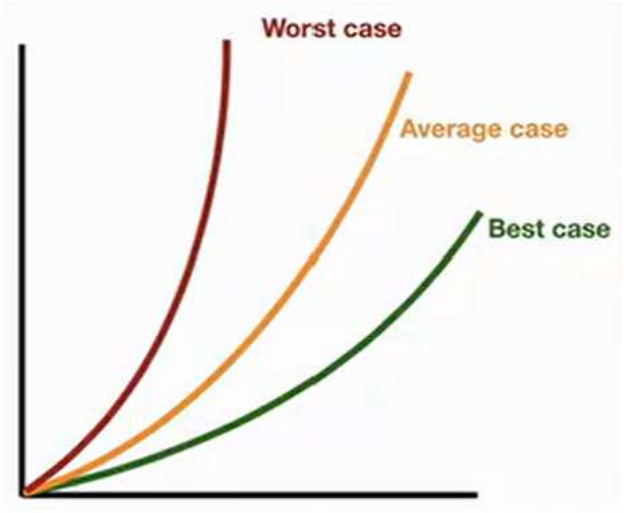
**Time complexity: A way of showing how the runtime of a function increases as the size of the input increases.**



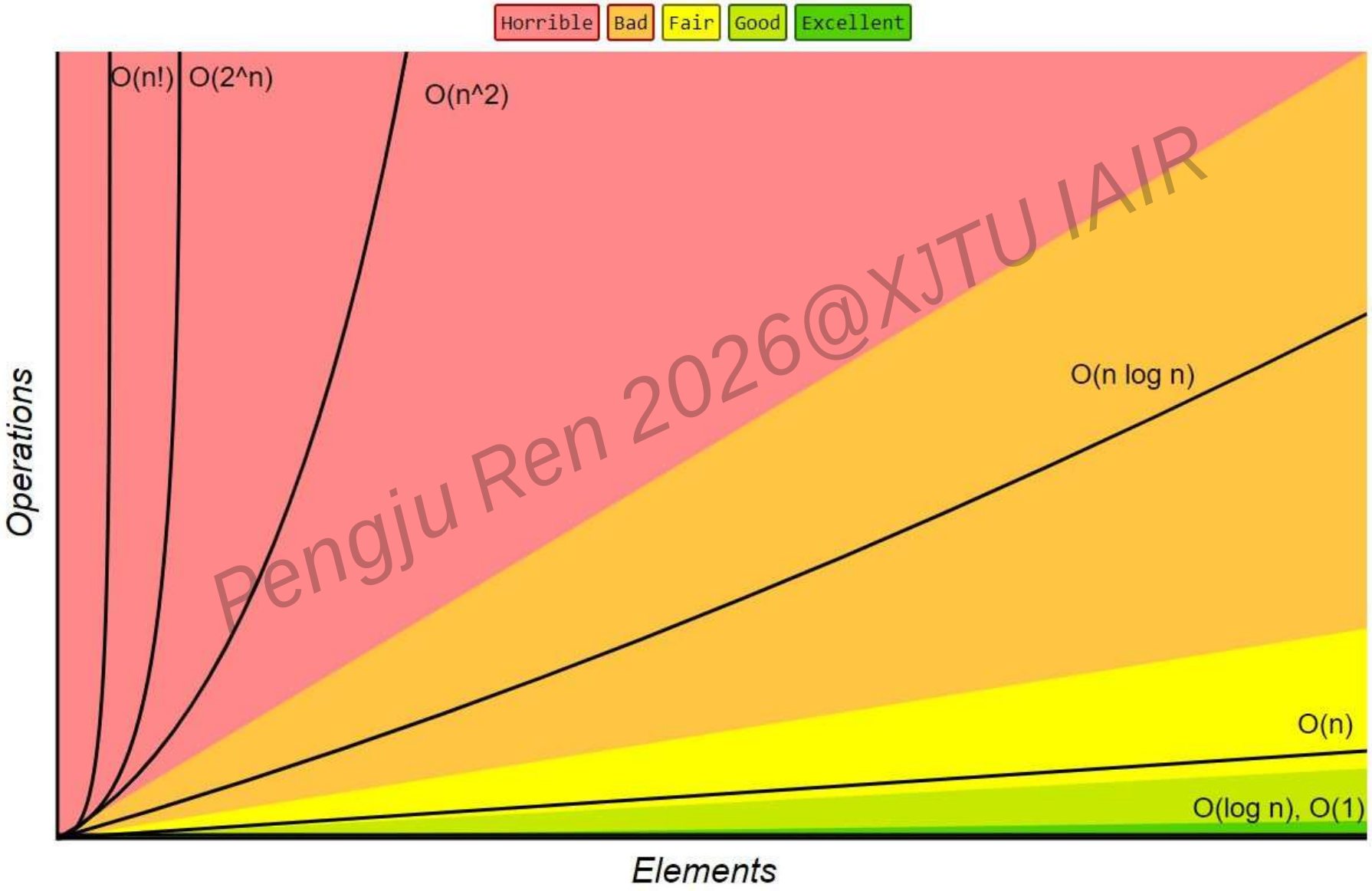
**$O(\cdot)$: It is a complexity that is going to be *less or equal to the worst case (Upper bound)***

**$\Omega(\cdot)$: It is a complexity that is going to be *at least more than the best case (Lower bound)***

**$\Theta(\cdot)$ *:* It is a complexity that is within bounds of the worst and the best case (Average bound)**



# Big-O Complexity Chart



# Runtime Complexities (Non Dominant Terms)

| Complexity | Name | Sample |
|---|---|---|
| $O(1)$ | Constant | *Operation not dependent on the input size (A simple add numbers function)* |
| $O(\log n)$ | Logarithmic | *Each iteration the problem size is reduced by half (Find an element in sorted array)* |
| $O(n)$ | Linear | *Iterate through numbers from 1 to n* |
| $O(n \log n)$ | N-log-n | *Sorting* |
| $O(n^2)$ | Quadratic | *Nested Loops* |
| $O(n^3)$ | Cubic | *Deeper nested Loops* |
| $O(2^n)$ | Exponential | *Double recursion in Fibonacci* |
| $O(n!)$ | Factorial | |

Excellent/Best
Good
Fair
Not Bad
Bad
Too Bad/Horrible



```
def multi_numbers(n):
    return n*n
```

```
def print_items(n):
    for i in range(n):
        print i
```

```
def print_items(n):
    for i in range(n):
        for j in range(n):
            print (i,j)
```

```
def recurisiveFib(n):
    if (n<2): return n;
    return recurisiveFib(n-1)+recurisiveFib(n-2)

print(recurisiveFib(10))
```

**Time Complexity: ?**

# Runtime Complexities (Non Dominant Terms)

| Complexity | Name | Sample |
|---|---|---|
| $O(1)$ | Constant | *Calculation is not dependent on the input size (A simple add numbers function)* |
| $O(\log n)$ | Logarithmic | *For each iteration the problem size is reduced by half (Find an element in sorted array)* |
| $O(n)$ | Linear | *Loop through numbers from 1 to n* |
| $O(n \log n)$ | N-log-n | *Sorting* |
| $O(n^2)$ | Quadratic | *Nested Loops* |
| $O(n^3)$ | Cubic | *Deeper nested Loops* |
| $O(2^n)$ | Exponential | *Double recursion in Fibonacci* |
| $O(n!)$ | Factorial | |

```
def print_items(a,b):
    for i in range(a):
        print i
    for j in range(b):
        print j
```

-If your algorithm is in the form *"do this, then when you are all done, do that"* then you **add** the runtimes

```
def print_items(a,b):
    for i in range(a):
        for j in range(b):
            print (i,j)
```

-If your algorithm is in the form *"do this for each time you do that"* then you **multiply** the runtimes

Time Complexity: ?

# Runtime Complexities (Non Dominant Terms)

| Complexity | Name | Sample |
|---|---|---|
| $O(1)$ | Constant | *Calculation is not dependent on the input size (A simple add numbers function)* |
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| $O(n!)$ | Factorial | |



```
def findBiggestNumber(simpleArray):
    biggestNumber = sampleArray[0]                          ..... O(1)
    for index in range (1, len(sampleArray)):               ..... O(n)
        if sampleArray[index] > biggestNumber:              ..... O(1)
            biggestNumber = sampleArray[index]              ..... O(1)
    print(biggestNumber)                                    ..... O(1)
```

O(1) + O(n) + O(1) = O(n)

O(n) + O(1) = O(n)

O(1) + O(1) = O(1)

**Time Complexity: ?**

# Runtime Complexity (Quiz)

```
for(i=0; i<n; i+2){
   Doing Something // O(1)
}
```
$O\left(\frac{n}{2}\right) = O(n)$

```
for(i=1; i<n; i=ix2){
   Doing Something // O(1)
}
```
$O(\log_2 n) = O(\log n)$

```
for(i=0; i<n; i+k){
   Doing Something // O(1)
}
```
$O\left(\frac{n}{k}\right) = O(n)$

```
for(i=1; i<n; i=ixk){
   Doing Something // O(1)
}
```
$O(\log_k n) = O(\log n)$

```
for(i=0; i<n; i++){
   for(j=0; j<i; j++){
     Doing Something // O(1)
   }
}
```
$\frac{n(n+1)}{2} \Rightarrow O(n^2)$

```
for(i=0; i<n; i++){
   for(j=1; j<n; j=jx2){
     Doing Something // O(1)
   }
}
```
$O(n \log n)$

```
p=0
for(i=0; p<n; i++){
   p = p+i;
}
```
$\frac{k(k+1)}{2} \geq n \Rightarrow O(\sqrt{n})$

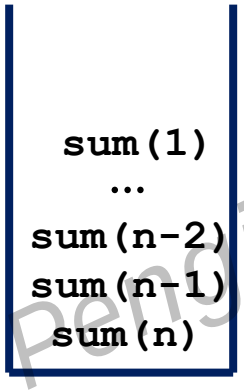
```
p=0
for(i=0; p<n; i++){
   p = p+ixi; Doing Something // O(1)
}
```
$\frac{k(k+1)(2k+1)}{6} \geq n \Rightarrow O(\sqrt[3]{n})$

```
p=0
for(i=0; i<n; i=i*2){
   p++;
   for(j=0; j<p; j=j*2){
       Doing Something // O(1)
   }
}
```

# Space Complexity

**How many bytes consumed during execution ?**

```
def sum(n):
    if n <=0:
       return 0
    return n+sum(n-1)
```




Stack

Space Complexity: $O(n)$

```
def pair_sum_sequence(n):
    total = 0
    for i in range(n):
        total = total + pair_sum(i, i+1)
    return total

def pair_sum(a,b):
    return a+b
```

Space Complexity: ?

# Example of Algorithms (diff implementation diff complexity)

- **Three-way Set Disjointness**

```
def disjoint1(A, B, C):
 """Return True if there is no element common to all three lists."""
for a in A:
    for b in B:
        for c in C:
            if a == b == c: return False # we found a common value
return True                              # if we reach this, sets are disjoint
```

**Time Complexity:** $O(n^3)$

```
def disjoint2(A, B, C):
"""Return True if there is no element common to all three lists."""
for a in A:
    for b in B:
        if a == b:              # only check C if we found match from A and B
            for c in C:
                if a == c:      # (and thus a == b == c)
                    return False   # we found a common value
return True                        # if we reach this, sets are disjoint
```

**Time Complexity:** $O(n^2)$

**At the worst case, disjoint2() is with same complexity as disjoint1()**

# Example of Algorithms (diff implementation diff complexity)

- **Element Uniqueness**

```
def unique1(S):
"""Return True if there are no duplicate elements in sequence S."""
  for j in range(len(S)):
      for k in range(j+1, len(S)):
          if S[j] == S[k]: return False # found duplicate pair
return True                             # if we reach this, elements were
```

**Time Complexity:** $O(n^2)$

```
def unique2(S):
"""Return True if there are no duplicate elements in sequence S."""
  temp = sorted(S)                      # create a sorted copy of S
     for j in range(1, len(temp)):
         if S[j-1] == S[j]: return False # found duplicate pair
return True                             # if we reach this, elements were
```

**Time Complexity:** $O(n \log n)$

# The Motivation of Hashing

**It is time efficient in case of SEARCH operation**

| Data Structure | Search | Insertion | Deletion |
|---|---|---|---|
| Array (Python List) | $O(n)$ | $O(n)$ | $O(n)$ |
| Linked List | $O(n)$ | $O(1)$ | $O(1)$ |
| Tree | $O(\log n)$ | $O(\log n)$ | $O(\log n)$ |
| Hashing | $\boldsymbol{O(1)/O(n)}$ | $\boldsymbol{O(1)/O(n)}$ | $\boldsymbol{O(1)/O(n)}$ |



- On average Insertion/Deletion/Search operations take O(1) time
- When hash function is not good enough, they takes O(n) time

# Sorting Algorithms

# What is Sorting ?

**By definition sorting refers to arranging data in a particular format: either *ascending* or *descending***

- **Bubble Soring**
- **Selection Sorting**
- **Insertion Sorting**
- **Bucket Sorting**
- **Merge Sorting**
- **Quick Sorting**
- **Heap Sorting**

# Categories of Sorting ?

**Space used**

**In place sorting:** Sorting algorithms which *does not require* any extra space for sorting (e.g. *Bubble Sorting*)

**Out place sorting:** Sorting algorithms which requires *an extra space* for sorting (e.g. *Merge Soring*)

**Stability**

**Stable sorting:** contents *does not change* the sequence of similar content in which they appear (e.g. *Insertion Sorting*)

**Unstable sorting:** contents *might change* the sequence of similar content in which they appear (e.g. *Quick Sorting*)

# Stability

| Unsorted data | |
|---|---|
| Name | Age |
| David | 8 |
| Green | 8 |
| Emma | 7 |
| Fendy | 7 |
| Bob | 7 |
| Candy | 8 |
| Helen | 8 |
| Andy | 8 |

| Sorted by Name | |
|---|---|
| Name | Age |
| Andy | 8 |
| Bob | 7 |
| Candy | 8 |
| David | 8 |
| Emma | 7 |
| Fendy | 7 |
| Green | 8 |
| Helen | 8 |

| Sorted by Age | |
|---|---|
| Name | Age |
| Bob | 7 |
| Emma | 7 |
| Fendy | 7 |
| Andy | 8 |
| Candy | 8 |
| David | 8 |
| Green | 8 |
| Helen | 8 |

**Stable sorting**

| Sorted by Age | |
|---|---|
| Name | Age |
| Emma | 7 |
| Bob | 7 |
| Fendy | 7 |
| David | 8 |
| Andy | 8 |
| Green | 8 |
| Candy | 8 |
| Helen | 8 |

**Unstable sorting**

# Bubble Soring

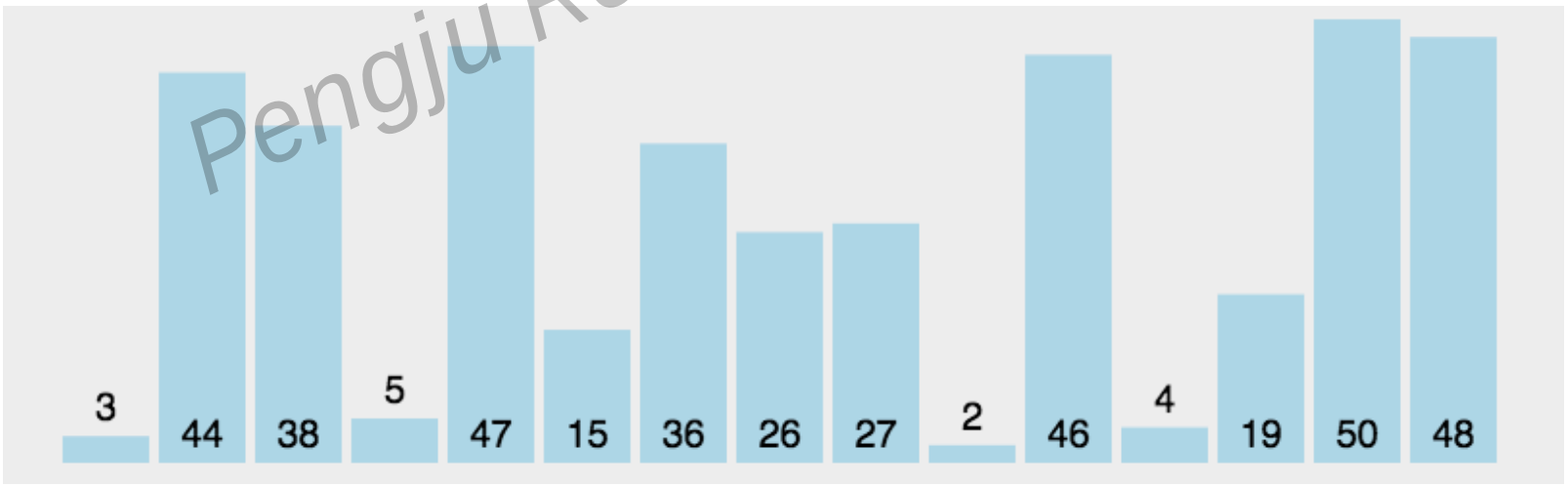
**Repeatedly compare each pair of adjacent items and swap them if they are in the wrong order**

```
def bubble_sort(arr):
    n = len(arr)
    for i in range(n):
      for j in range(0, n-i-1):
        if arr[j] > arr[j+1]:
                arr[j], arr[j+1] = arr[j+1], arr[j]
    return arr
```

# Selection Soring

**Repeatedly find the minimum element and move it to the sorted part of array to make unsorted part sorted**

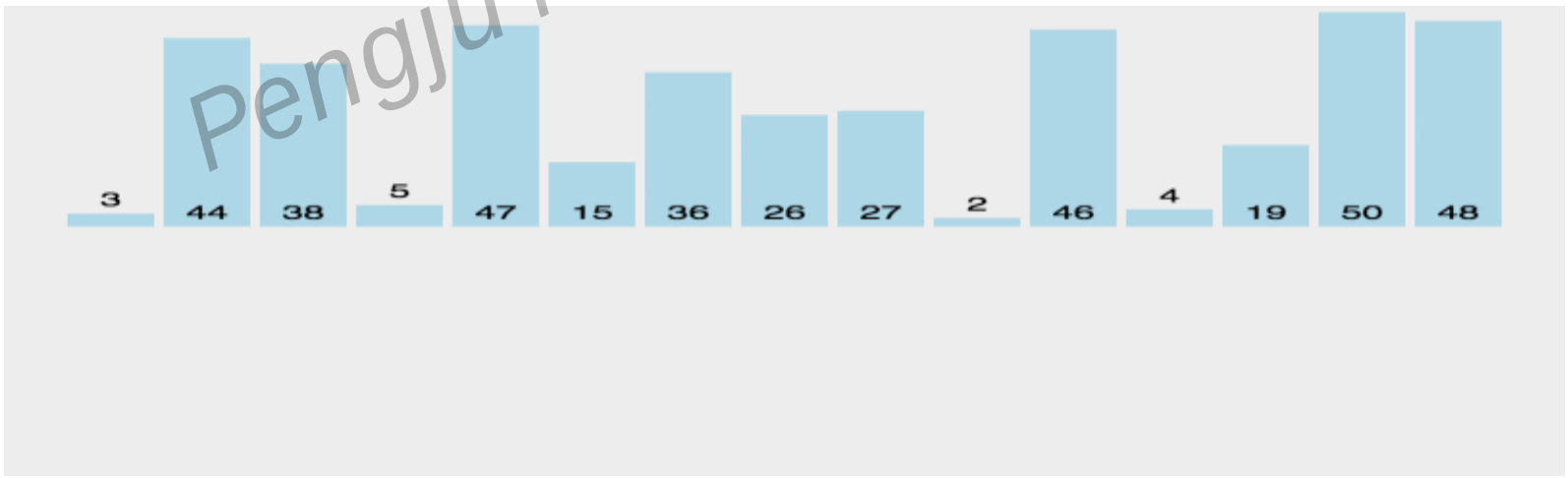
```
def selection_sort(arr):
    n = len(arr)
    for i in range(n):
        min_idx = i
        for j in range(i+1, n):
            if arr[j] < arr[min_idx]:
                min_idx = j
        arr[i], arr[min_idx] = arr[min_idx], arr[i]
    return arr
```



# Insert Soring

**Divide the array into two part, taking first element from unsorted array and find its correct position in sorted array, repeat until unsorted array is empty**

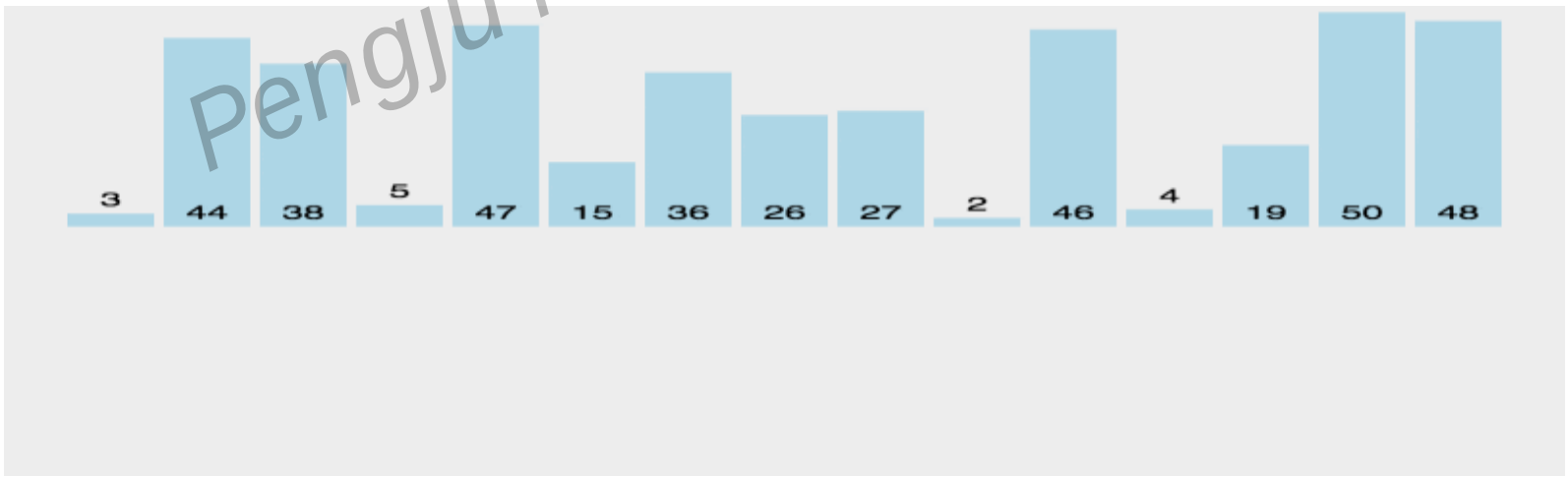
```
def insertion_sort(arr):
    n = len(arr)
    for i in range(1, n):
        key = arr[i]
        j = i-1
        while j >= 0 and key < arr[j]:
            arr[j+1] = arr[j]
            j -= 1
        arr[j+1] = key
    return arr
```



# Insert Soring

**Divide the array into two part, taking first element from unsorted array and find its correct position in sorted array, repeat until unsorted array is empty**

```
def insertion_sort(arr):
    n = len(arr)
    for i in range(1, n):
        key = arr[i]
        j = i-1
        while j >= 0 and key < arr[j]:
            arr[j+1] = arr[j]
            j -= 1
        arr[j+1] = key
    return arr
```



# Bucket Sorting

**Create buckets and distribute elements of array into buckets, sort buckets individually, then merge buckets after sorting**

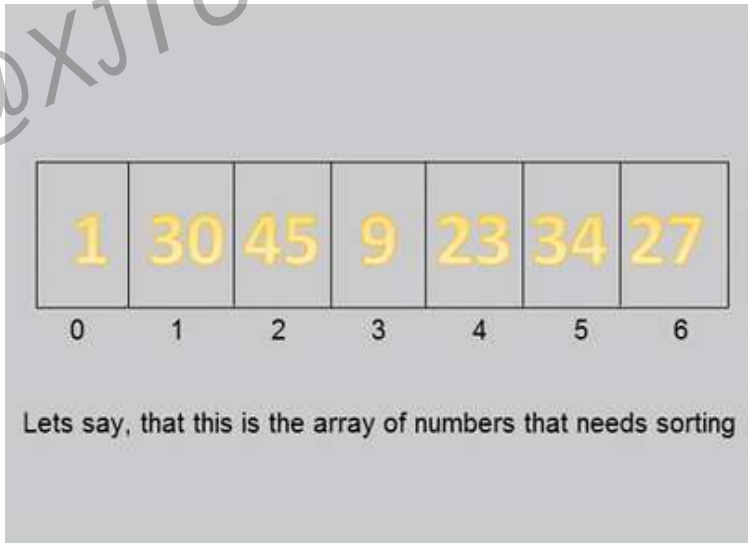
```
def bucket_sort(arr, bucket_size=4):
    if len(arr) == 0:
        return arr
    min_val, max_val = min(arr), max(arr)

    bucket_count = (max_val - min_val)
    buckets = [[] for _ in range(bucket_count)]

    for num in arr:
        idx = (num - min_val)
        buckets[idx].append(num)

    sorted_arr = []
    for bucket in buckets:
        insertion_sort(bucket)
        sorted_arr.extend(bucket)

    return sorted_arr
```



# Merger Sorting

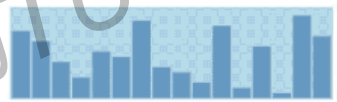
**Divide the array in two halves and keep halving recursively until they become one element, merge halves by sorting them**

```
def merge_sort(arr):
    if len(arr) <= 1:
        return arr

    mid = len(arr) // 2
    left = merge_sort(arr[:mid])
    right = merge_sort(arr[mid:])

return merge(left, right)
```

```
def merge(left, right):
    result = []
    i = j = 0
    while i<len(left) and j<len(right):
        if left[i] <= right[j]:
            result.append(left[i])
            i += 1
        else:
            result.append(right[j])
            j += 1
    result.extend(left[i:])
    result.extend(right[j:])
    return result
```



# Quick Sorting

**Selecting a *pivot element* from the array and partitioning the other elements into two sub-arrays, according to whether they are < or > than the pivot. Then sub-arrays are sorted recursively**

```
def quick_sort(arr):
    if len(arr) <= 1:
        return arr
    pivot = arr[len(arr)//2]
    left = [x for x in arr if x < pivot]
    middle = [x for x in arr if x == pivot]
    right = [x for x in arr if x > pivot]
    return quick_sort(left) + middle + quick_sort(right)
```

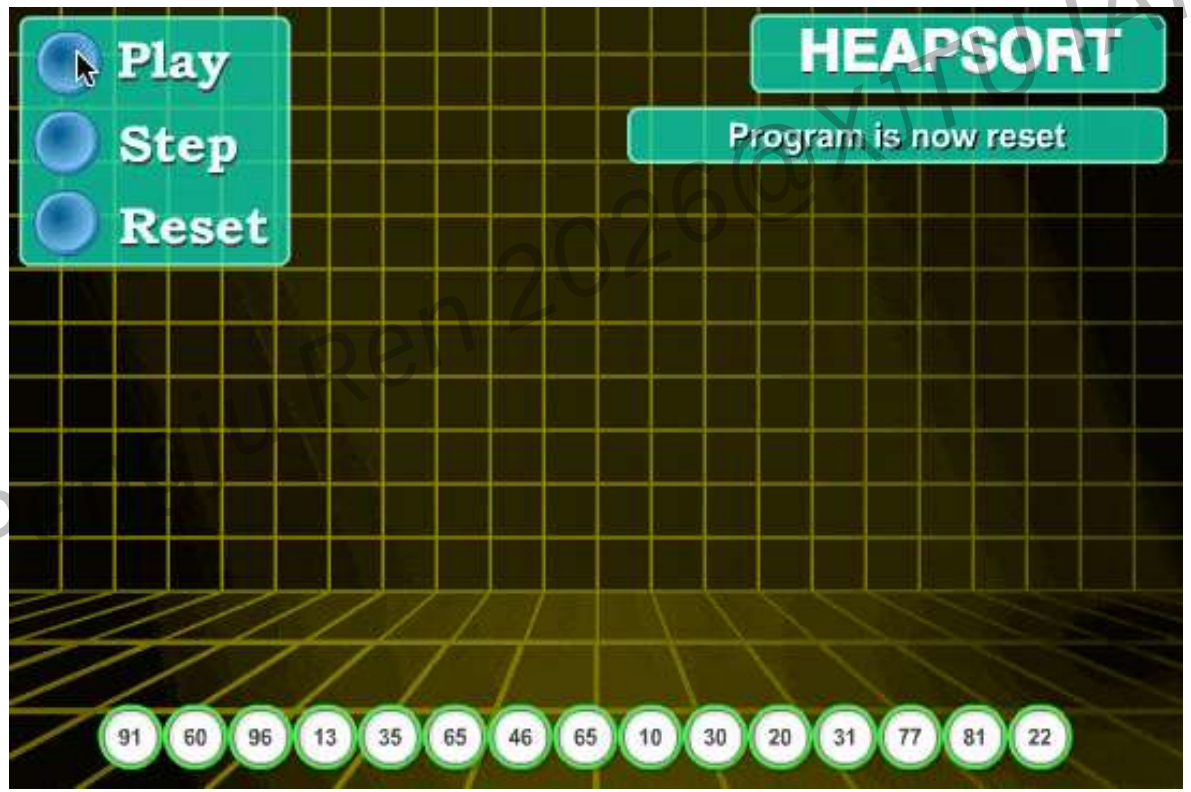


@五分钟学算法之快速排序

# Heap Sorting (Suited for array, not linked-list)

**Step1: Insert data to binary Heap Tree**

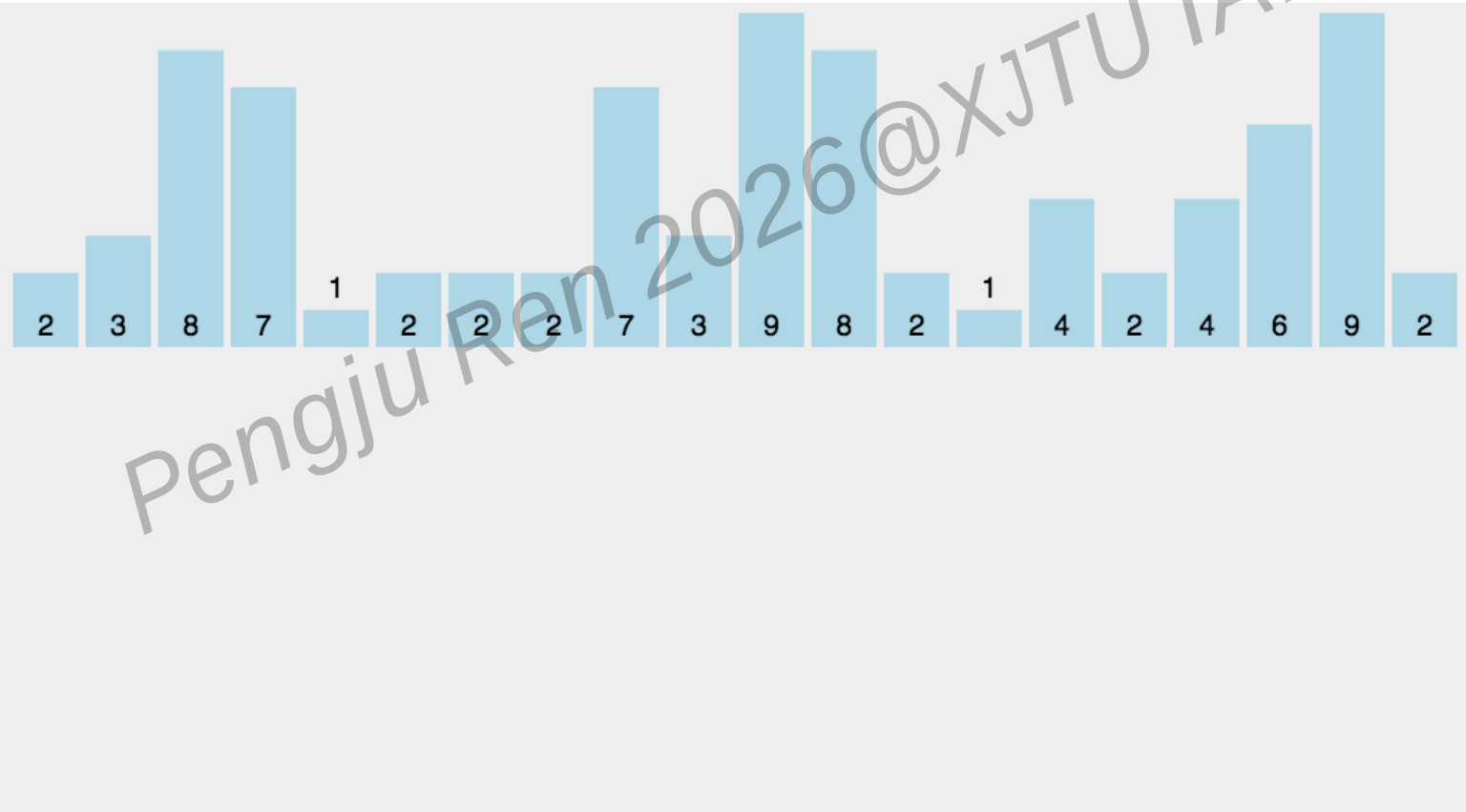
**Setp2: Extract data from Binary Heap**



# Count Sorting

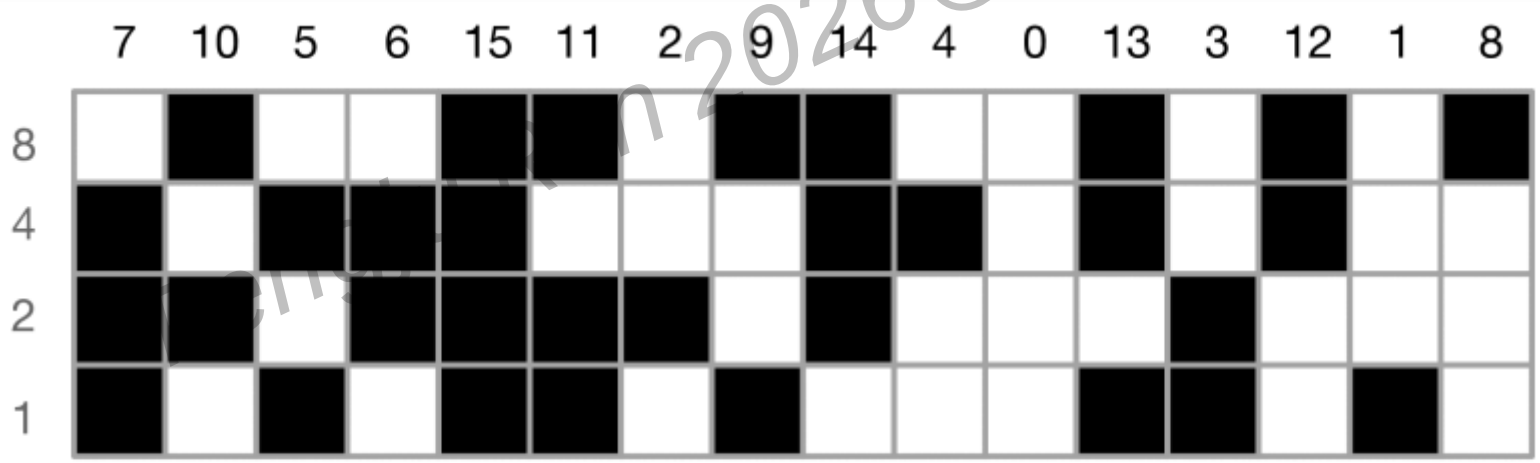
**Step1 : Find the maximum and minimum elements in the array to be sorted;**

**Step2 : Count the occurrences of each element with value i in the array and store the counts in the i-th element of array C**



# Radix Sorting

- **Step1: Determine the Maximum Digits**: Identify the maximum number of digits in the largest number in the list.
- **Sort by Each Digit**: Starting from the least significant digit, sort all numbers using a stable sorting algorithm like Counting Sort.
- **Repeat for Higher Digits**: Continue sorting for each digit until all place values are processed.



# Sorting Algorithms

| Name | Best Case | Time Complexity | Worst Case | Space Complexity | Stable |
|---|---|---|---|---|---|
| Bubble | $O(n)$ | $O(n^2)$ | | $O(1)$ | Y |
| Selection | | $O(n^2)$ | | $O(1)$ | N |
| Insertion | $O(n)$ | $O(n^2)$ | | $O(1)$ | Y |
| Bucket | $O(n)$ | $O(n \log n)$ | | $O(1)$ | Y |
| Merge | | $O(n \log n)$ | | $O(n)$ | Y |
| Quick | | $O(n \log n)$ | $O(n^2)$ | $O(n)$ | N |
| Heap | | $O(n \log n)$ | | $O(1)$ | N |
| Count | | $O(n)$ | | $O(n)$ | Y |
| Radix | | $O(n)$ | | $O(n)$ | Y |

# Summary

- **Hashing**: It is a function that can be used to map of arbitrary size to data of fixed size
- **Algorithm Complexity**: Running time and Space usage are used to analysis the efficiency of particular algorithm (**Time/Space Complexity**)
- **Sorting**: Master various sorting algorithms and perform comparisons from different perspectives.